1. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 5 \\ 0 & 2 & -1 & 3 & -2 \\ 0 & -4 & 2 & -6 & 4 \\ 0 & 6 & -3 & 8 & 1 \end{bmatrix}$$

   (a) (2 points) Reduce A to its echelon form B.

   (b) (6 points) Find a basis from the columns of A that form a basis for the column space of A.

   (c) (7 points) Find a basis from the rows of A that form a basis for the row space of A.

2. (15 points) The matrix Q is given below, along with the echelon forms of $(Q|b)$ and $(Q|0)$ are given below:

$$Q = \begin{pmatrix} 2 & 3 & 3 & 1 & 0 & 7 \\ 3 & 1 & 8 & 2 & 4 & 4 \\ 1 & 2 & 1 & 0 & -1 & 4 \\ 3 & 1 & 8 & 1 & 3 & 3 \\ -2 & -1 & -5 & 1 & 0 & -1 \end{pmatrix}. \qquad (Q|0) \rightarrow \begin{pmatrix} 1 & 0 & 3 & -1 & 0 & -1 & 0 \\ 0 & -7 & 7 & 1 & 8 & -13 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

   Find a basis for the nullspace of Q. Extend this basis to an orthogonal basis for R^5.

3. Please solve the following questions giving suitable justifications.

   (5 points)(a) A is a 3×3 matrix. B is a 3×4 matrix. is it possible that the columns of AB are linearly independent.

   (5 points) (b) Let V be a vector space and d be a metric on it. Does d always arise from a norm ?

4. Let $A_{m\times n}$ be a given matrix over the real numbers.

   (a) (5 points) Let B be obtained from A by sweeping out the l-th column using the element a_{kl} as the pivot. Show that $\{B_{i_1*}, B_{i_l*}, \ldots B_{i_p*}\}$ are independent if and only if $\{A_{i_1*}, A_{i_l*}, \ldots A_{i_p*}\}$ are independent then $k \in \{i_1, \ldots i_p\}$.

   (b) (5 points) Let C be a matrix obtained from A by sweeping out the l-th column with a_{kl} as pivot and followed by a permutation π of the rows not involving the k-th row. Let D be a matrix obtained from A by a permutation π of the rows not involving the k-th row and followed by sweeping out the l-th column with a_{kl} as pivot. Show that $C = D$.